AP® CALCULUS AB
2007 SCORING GUIDELINES

Question 1

Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

$$\frac{20}{1+x^2} = 2 \text{ when } x = \pm 3$$

1 : correct limits in an integral in (a), (b), or (c)

(a) $$\text{Area} = \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right) dx = 37.961 \text{ or } 37.962$$

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(b) $$\text{Volume} = \pi \int_{-3}^{3} \left(\left(\frac{20}{1+x^2} \right)^2 - 2^2 \right) dx = 1871.190$$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(c) $$\text{Volume} = \frac{\pi}{2} \int_{-3}^{3} \left(\frac{1}{2} \left(\frac{20}{1+x^2} - 2 \right) \right)^2 dx$$
$$= \frac{\pi}{8} \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right)^2 dx = 174.268$$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

1A1

CALCULUS BC

SECTION II, Part A

Time—45 minutes

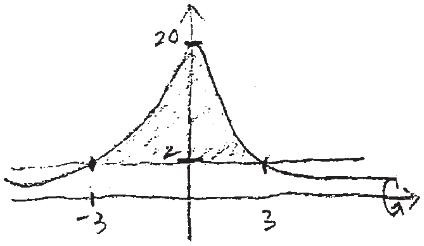
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

Area $= \int_{-3}^{3}\left(\frac{20}{1+x^2} - 2\right) dx$

$= 37.9618$



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Continue problem 1 on page

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Work for problem 1(b)

$R = \frac{20}{1+x^2}$ $r = 2$

$V = \pi \int_{-3}^{3} \left[\left(\frac{20}{1+x^2} \right)^2 - 4 \right] dx$

$= 1871.1901$

Work for problem 1(c)

$D = \frac{20}{1+x^2} - 2$

$r = \frac{\frac{20}{1+x^2} - 2}{2} = \frac{10}{1+x^2} - 1$

$V = \int_{-3}^{3} \frac{\pi \left(\frac{10}{1+x^2} - 1 \right)^2}{2} dx = \frac{\pi}{2} \int_{-3}^{3} \left(\frac{10}{1+x^2} - 1 \right)^2 dx$

$= 174.2685$

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1B

CALCULUS BC

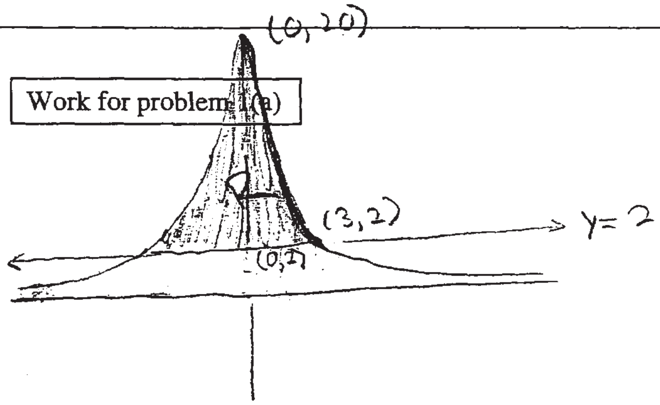
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)



~~$y = \frac{20}{1+x^2}$~~

~~$1+x^2 = \frac{20}{y}$~~

$$A_R = 2\int_0^3 \left[\left(\frac{20}{1+x^2}\right) - 2\right] dx$$

$$A_R = 37.962$$

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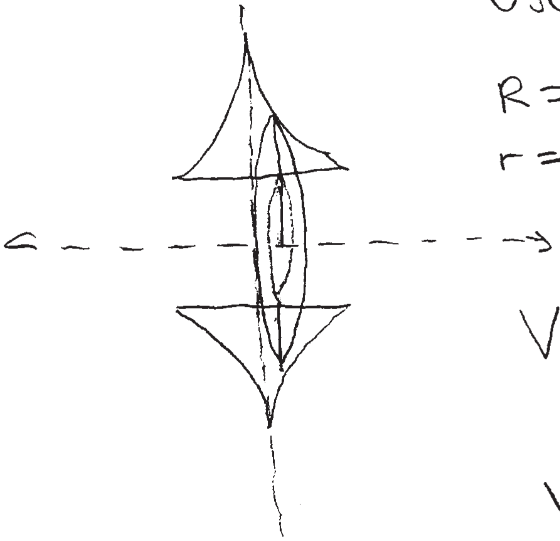
Continue problem 1

Work for problem 1(b)



Use washers method.

$R = \frac{20}{1+x^2}$

$r = 2$

$$V = 2\left[\pi \int_0^3 \left(\left(\frac{20}{1+x^2}\right)^2 - 2^2\right) dx\right]$$

$V = 1871.190$

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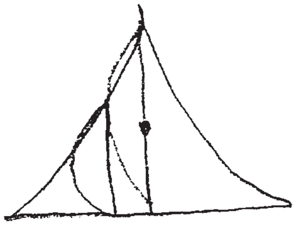
Work for problem 1(c)



$$r = \frac{\left(\frac{20}{1+x^2}\right)}{2} = \frac{10}{1+x^2}$$

$$V = 2\left[\frac{1}{2}\pi \int_0^3 \left(\frac{10}{1+x^2}\right)^2\right]$$

$V = 243.324$

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1C1

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

R shaded region bounded by $y = \frac{20}{1+x^2}$ & $y = 2$

Intesect at $(-3,2)$ & $(3,2)$

A of R =

$$\int_{-3}^{3} \left(\frac{20}{1+x^2} - 2 \right) dx$$

Area of shaded region R = $\boxed{37.962 \text{ units}^2}$

Continue problem 1

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Work for problem 1(b)

$$V = \pi \int_a^b \left(R(x) - r(x)\right)^2 dx$$

$$V = \pi \int_{-3}^{3} \left(\frac{20}{1+x^2} - 2\right)^2 dx$$

$$\boxed{V = 1394.148 \text{ units}^3}$$

Work for problem 1(c)

A of semicircle $\frac{1}{2}\pi r^2$

$$V : \int_{-3}^{3} \frac{1}{2}\pi r^2 \quad \text{where } r = \frac{20}{1+x^2}$$

$$V = \int_{-3}^{3} \frac{1}{2}\pi \left(\frac{20}{1+x^2}\right)^2$$

$$\boxed{V : 973.294 \text{ units}^3}$$

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Question 1

Overview

This problem presented students with a region bounded above by the graph of a function and below by a horizontal line. Because no picture was provided, students were expected to graph the function on their calculators or use their knowledge of rational functions to sketch the graph, and then identify the appropriate region from their graph. The points of intersection of the graph and the horizontal line could be found either algebraically or with the calculator. Students needed to find, in part (a), the area of the region; in part (b), the volume of the solid generated when the region was rotated about the *x*-axis; and in part (c), the volume of the solid above the region for which the cross sections perpendicular to the *x*-axis were semicircles.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: the region point, 2 points in part (a), 3 points in part (b), and no points in part (c). In part (a) the student sets up the definite integral using symmetry and earned the region point. The student has the correct integrand, which earned the first point in part (a). The answer is correct to three decimal places and earned the second point. In part (b) the student continues to use symmetry in defining the limits of integration. The correct integrand earned the first 2 points. The answer is correct to three decimal places and earned the third point. In part (c) the student did not earn any points because the radius is incorrect.

Sample: 1C
Score: 3

The student earned 3 points: the region point, 2 points in part (a), no points in part (b), and no points in part (c). In part (a) the student earned the region point by using the correct limits of integration. The student has the correct integrand, which earned the first point in part (a). The answer is correct to three decimal places and earned the second point. In part (b) the student did not earn any points because the washer method is not used correctly. In part (c) the student did not earn any points because the radius is incorrect.

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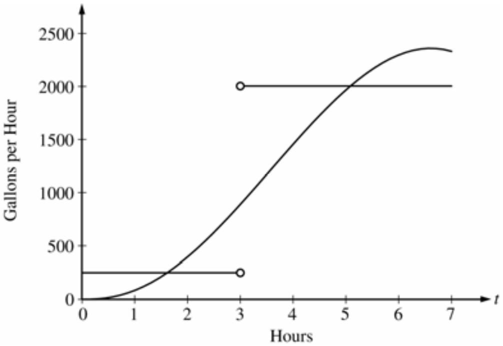
Question 2

The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is $f(t) = 100t^2 \sin\left(\sqrt{t}\right)$ gallons per hour for $0 \le t \le 7$.

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \le t < 3 \\ 2000 & \text{for } 3 < t \le 7 \end{cases} \text{ gallons per hour.}$$



The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

(a) How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.

(c) For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a) $\int_0^7 f(t)\,dt \approx 8264$ gallons

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

(b) The amount of water in the tank is decreasing on the intervals $0 \le t \le 1.617$ and $3 \le t \le 5.076$ because $f(t) < g(t)$ for $0 \le t < 1.617$ and $3 < t < 5.076$.

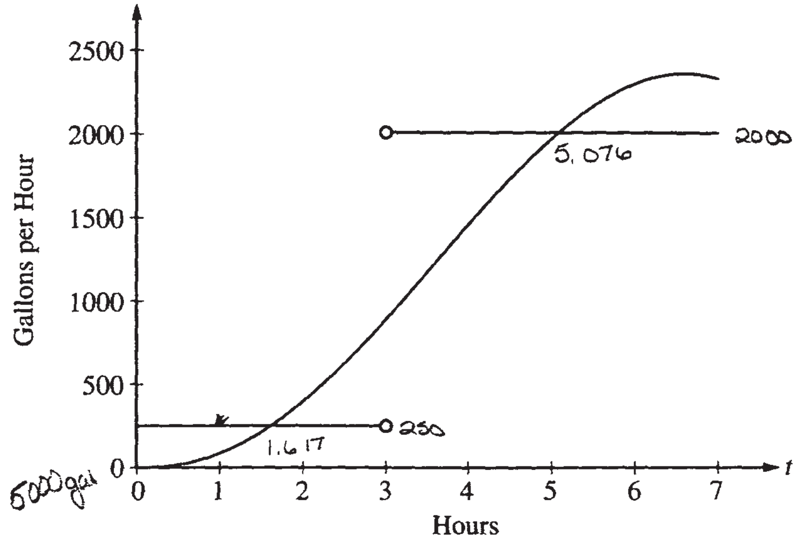
$$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$$

(c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0,\ 3,$ and 7.

t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t)\,dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t)\,dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.

$$5 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$$



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Work for problem 2(a)

$\int_0^7 f(t)dt$

8264 gallons

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Continue problem 2 on page 7.

Work for problem 2(b)

$r(t) = f(t) - g(t)$

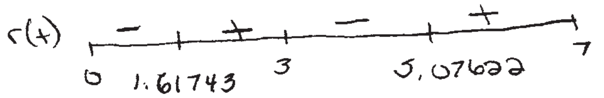
$r(t) = f(t) - 250$ for $0 \le t < 3$ $r(t) = f(t) - 2000$ for $3 < t \le 7$

$0 = f(t) - 250$

$t = 1.61743$

$0 = f(t) - 2000$

$t = 5.07622$



the amount of water in the tank is decreasing on (0, 1.61743) b/c r(t) that is defined for $0 \le t < 3$ is negative. It is also decreasing on (3, 5.07622) b/c r(t) that is defined for $3 < t \le 7$ is negative.

Work for problem 2(c)

Relative max at time t=3 b/c r(t) changes from (+) to (−)

$5000 + \int_0^3 r(t)dt =$

5126.591

(0, 5000)

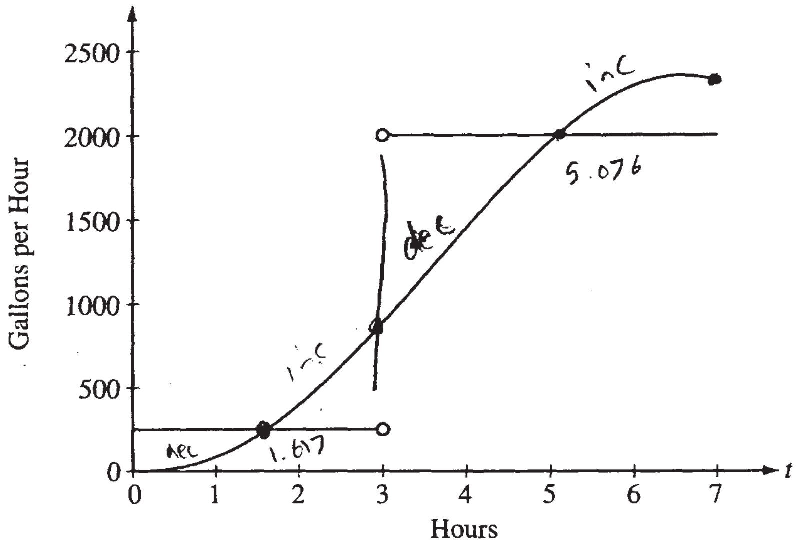
(3, 5126.591)

(7, 4513.807)

$5000 + \int_0^3 r(t)dt + \int_3^7 r(t)dt =$

4513.807

The amount of water in the tank is greatest at time t=3 which is a relative max for the function of the amount of water in the tank. At the boundaries of the interval, the amount of water in the tank is equal to 5000, at t=0, and 4514, at t=7. At t=3, the amount of water in the tank is 5127 gallons, which is greater than the values at the boundaries.



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Work for problem 2(a)

$t=0$, 5000 gals

a) $\int_0^7 100t^2 \sin(\sqrt{t})\,dt = \boxed{8264 \text{ gallons}}$

Continue problem 2 on page 7.

Work for problem 2(b)

b) water is decreasing (0, 1.617), because $g(t) > f(t)$ on that interval.

water is decreasing (3, 5.076), because $g(t) > f(t)$ on that interval

Work for problem 2(c)

$t=0$, 5000 gals

c) $h(t) = f(t) - g(t)$

$h(7) = f(7) - g(7)$

$100(7)^2 \sin\sqrt{7} - 2000$

$5000 + \int_0^7 100t^2 \sin\sqrt{t} - 2000 \, dt = -736.19$

$5000 + \int_0^0 100t^2 \sin\sqrt{t} - 2000 \, dt = 5000$

$5000 + \int_0^3 100t^2 \sin\sqrt{t} - 2000 \, dt = -123.409$

$h(0) = 5000$

$h(7) = -736.19$

$h(3) = -123.409$

At $t=0$, the amount of water is greatest

At $t=0$, the amount of water is 5000 gallons

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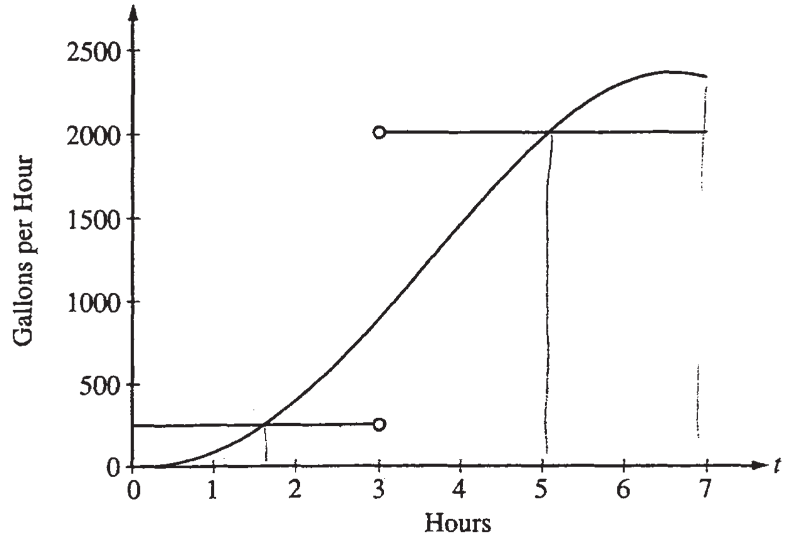

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Work for problem 2(a)

$\int_0^7 100t^2 \sin(\sqrt{t})\,dt \approx 8264$ gallons

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Continue problem 2 on page 7.

Work for problem 2(b)

The amount in the tank is decreasing on the intervals (0, 1.617) ∪ (1.617, 5.076) because on those intervals the rate of water entering is less than the rate of the water leaving.

Work for problem 2(c)

At time t=7 because the water entering is greater than the water leaving is at a max for the interval.

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AP® CALCULUS AB
2007 SCORING COMMENTARY

Question 2

Overview

This problem presented students with two functions that modeled the rates, in gallons per hour, at which water entered and left a storage tank. The latter function was piecewise-constant. Graphs of each function were also provided. In part (a) students had to use a definite integral to find the total amount of water that entered the tank over a given time interval. Part (b) measured their abilities to compare the two rates to find, with justification, the time intervals during which the amount of water in the tank was decreasing. This could be determined directly from the graphs and the information given about the points of intersection, but students needed to be able to handle the point of discontinuity in the piecewise-defined function. Part (c) asked for the time at which the amount of water was at an absolute maximum and the value of this maximum amount to the nearest gallon. Again, dealing with the critical point at the discontinuity was an important part of the analysis, as was using the net rate of change during the first three hours and during the last four hours to compute the total amount of water in the tank at $t = 3$ and $t = 7$, respectively.

Sample: 2A
Score: 9

The student earned all 9 points. Note that in part (b), in the presence of the correct numerical values in reported intervals, errors in the use of open, closed, or half-open interval notation were ignored, and the student earned the interval point. The student defines $r(t)$ in part (b) and that definition may be used in part (c).

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), and 2 points in part (c). In part (a) the student gives the correct integral and the correct answer and earned both the integral and answer points. In part (b) the student gives the correct intervals and a correct reason and earned both the interval and reason points. In part (c) the student considers $t = 0,$ $t = 3,$ and $t = 7$ as candidates for the absolute maximum with the evaluation of the three integrals presented. The student therefore considers $t = 3$ a candidate and earned the first point. The student presents the integrand $f(t) - g(t)$ in an integral and earned the second point. The incorrect use of the rule for $g(t)$ results in incorrect values for the amount of water at time $t = 3$ and $t = 7,$ so the third and fourth points in part (c) were not earned; and the student presents an incorrect conclusion, so the fifth point was not earned.

Sample: 2C
Score: 3

The student earned 3 points: 2 points in part (a), 1 point in part (b), and no points in part (c). In part (a) the student gives the correct integral and the correct answer and earned both the integral and answer points. In part (b) the student reports an incorrect left endpoint on the second interval, so the interval point was not earned. The student provides correct reasoning and earned the reason point. In part (c) the student never considers $t = 3$ as a candidate for the absolute maximum, so the first point was not earned. No integrand in an integral is presented, and the amounts of water at $t = 3$ and $t = 7$ are not calculated. The second, third, and fourth points in part (c) were not earned, and the student presents an incorrect conclusion, so the fifth point was not earned.

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Question 3

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

(c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

(d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

(a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$ $h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$ Since $h(3) < -5 < h(1)$ and h is continuous, by the Intermediate Value Theorem, there exists a value r, $1 < r < 3$, such that $h(r) = -5$.	$2: \begin{cases} 1: h(1) \text{ and } h(3) \\ 1: \text{conclusion, using IVT} \end{cases}$
(b) $\dfrac{h(3) - h(1)}{3 - 1} = \dfrac{-7 - 3}{3 - 1} = -5$ Since h is continuous and differentiable, by the Mean Value Theorem, there exists a value c, $1 < c < 3$, such that $h'(c) = -5$.	$2: \begin{cases} 1: \dfrac{h(3) - h(1)}{3 - 1} \\ 1: \text{conclusion, using MVT} \end{cases}$
(c) $w'(3) = f(g(3)) \cdot g'(3) = f(4) \cdot 2 = -2$	$2: \begin{cases} 1: \text{apply chain rule} \\ 1: \text{answer} \end{cases}$
(d) $g(1) = 2$, so $g^{-1}(2) = 1$. $\left(g^{-1}\right)'(2) = \dfrac{1}{g'\left(g^{-1}(2)\right)} = \dfrac{1}{g'(1)} = \dfrac{1}{5}$ An equation of the tangent line is $y - 1 = \frac{1}{5}(x - 2)$.	$3: \begin{cases} 1: g^{-1}(2) \\ 1: \left(g^{-1}\right)'(2) \\ 1: \text{tangent line equation} \end{cases}$

3A1

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	−4	4	2
4	−1	3	6	7

Work for problem 3(a)

Differentiability implies continuity so f and g are also continuos for all real numbers. Because f and g are continuous, $h(x)$ is also continuous for all real number. Because h is continuous and $h(3) = -7$ and $h(1) = 3$, $-7 = h(3) < -5 = h(r) < 3 = h(1)$, so a value of r where $h(r) = -5$ is guaranteed by the Intermediate Value Theorem.

Work for problem 3(b)

$h(x)$ is continuous and differentable on all real numbers because g and f are continuous and differentable for all real numbers.

Because $h(x)$ is continuous on $[1,3]$ and differentable on $(1,3)$ and $\frac{h(3)-h(1)}{3-1} = \frac{-7-3}{2} = -5$, the Mean Value Theorem guarantees that there is a value of c where $h'(c) = -5$

Continue problem 3 on page 9.

3A$_2$

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Work for problem 3(c)

$u = g(x)$

$u' = g'(x)$

$w'(x) = g'(x) \cdot \frac{d}{du} \int_1^u f(t)\,dt$

by the 2nd Fundamental theorem

$w'(x) = g'(x) \cdot f(u) = g'(x) \cdot f(g(x))$

$w'(3) = g'(3) \cdot f(g(3))$

$w'(3) = 2 \cdot f(4)$

$w'(3) = -2$

Work for problem 3(d)

$^{-1}(2, \ )$

$(\ , 2)$

$\frac{d}{dx}(g^{-1}(2)) = \frac{1}{g'(1)} = \frac{1}{5} = m$

$1 = g(x) \quad x = 2$

$g^{-1}(2) = 1$

$y - 1 = \frac{1}{5}(x - 2)$

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

3B1

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

Work for problem 3(a)

$h(1) = f(g(1)) - 6$
$= f(2) - 6$
$= 9 - 6$
$= 3$

$h(3) = f(g(3)) - 6$
$= f(4) - 6$
$= -1 - 6$
$= -7$

according to Rolle's Theorem, if $h(a) = c$ and $h(b) = d$ and g is on the interval $c < g < d$, then there must be a value r that exists on the interval $a < r < b$ for which $h(r) = g$

$h(1) = 3$ and $h(3) = -7$. -5 is on the interval $-7 < -5 < 3$, ∴ there must be a value r for which $h(r) = -5$.

Work for problem 3(b)

$$\frac{h(3) - h(1)}{3-1} = \frac{-7-3}{2} = -5$$

according to the Mean Value theorem, if $\frac{F(b) - F(a)}{b - a} = m$, then there must be a value c on the interval $a < c < b$ for which $f(c) = m$

$\frac{h(3) - h(1)}{3-1} = -5$ ∴ there must be a value c on the interval $1 < c < 3$ for which $h'(c) = -5$.

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Continue problem 3 on page 9.

$3B_2$

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Work for problem 3(c)

$w'(x) = g'(x)\,(f(g(x)))$

$w'(3) = g'(3)(f(g(3)))$

$= g'(3)f(4)$

$= (2)(-1)$

$w'(3) = -2$

Work for problem 3(d)

$g(x)$: $y = 2$

$x = 1$

$g^{-1}(x)$ $x = 2$

$y = 1$

$m = 1$

$1 = (1)(2) + c$

$-1 = c$

$y = x - 1$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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3C1

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	−4	4	2
4	−1	3	6	7

Work for problem 3(a)

$h(1) = 3$ and $h(3) = -7$ and since $h'(x)$ is strictly decreasing over this interval, there must be some value, r, over the interval $(1, 3)$ where $h(x) = -5$.

Work for problem 3(b)

$h'(x) = f'(g(x))(g'(x))$

Then, since $f'(x)$ is negative over this interval, the slope of $h'(x)$ must therefore be negative at some point.

Continue problem 3 on page 9.

$3C_2$

Work for problem 3(c)

$$w'(x) = f(g(x))\,g'(x)$$

$$f(g(3)) \times g'(3)$$

$$-1 \times 2$$

$$\boxed{-2}$$

Work for problem 3(d)

$$y = \frac{1}{5}x + b$$

$$3 = \frac{1}{5}(2) + b$$

$$3 = \frac{2}{5} + b$$

$$\frac{5}{15} - \frac{6}{15} = b$$

$$\frac{-1}{15} = b$$

$$y = \frac{1}{5}x - \frac{1}{15}$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

AP® CALCULUS AB
2007 SCORING COMMENTARY

Question 3

Overview

This problem presented students with a table of selected values of functions f and g, and their first derivatives. A third function h was defined in terms of the composition of f and g. Parts (a) and (b) assessed students' abilities to use the chain rule, the Intermediate Value Theorem, and the Mean Value Theorem to explain why there must be values r and c in the interval $(1, 3)$ where $h(r) = -5$ and $h'(c) = -5$. In part (c) students were given a function w defined in terms of a definite integral of f where the upper limit was $g(x)$. They had to use the Fundamental Theorem of Calculus and the chain rule to calculate the value of $w'(3)$. Part (d) asked students to write an equation for a line tangent to the graph of the inverse function of g at a given value of x. In all parts of this problem students had to use appropriate values from the given table to do their calculations.

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 6 points: 1 point in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student correctly identifies $h(1)$ and $h(3)$ for the first point but mistakenly identifies Rolle's Theorem and thus did not earn the second point. In part (b) the student calculates the difference quotient and applies the Mean Value Theorem to support the conclusion and earned both points. In part (c) the student earned both points by correctly applying the Fundamental Theorem of Calculus and the chain rule and by correctly evaluating the function. In part (d) the student correctly declares $g^{-1}(2)$ and earned the first point. The student did not earn the second point for $\left(g^{-1}\right)'(2)$, nor did the student declare a value for $\left(g^{-1}\right)'(2)$, so the student was not eligible for the third point.

Sample: 3C
Score: 4

The student earned 4 points: 1 point in part (a), no points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student correctly identifies $h(1)$ and $h(3)$ for the first point but does not apply the hypotheses of the Intermediate Value Theorem to support the conclusion and did not earn the second point. In part (b) the student does not calculate the difference quotient and was not eligible for either point. In part (c) the student earned both points by correctly applying the Fundamental Theorem of Calculus and the chain rule and by correctly evaluating the function. In part (d) the student did not earn the first point for $g^{-1}(2)$. The student correctly uses $\left(g^{-1}\right)'(2)$ in the tangent line equation and earned the second point. The student does not declare a value for $g^{-1}(2)$, so was not eligible for the third point.

AP® CALCULUS AB
2007 SCORING GUIDELINES

Question 4

A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi$.

(a) Find the time t at which the particle is farthest to the left. Justify your answer.

(b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

(a) $x'(t) = -e^{-t}\sin t + e^{-t}\cos t = e^{-t}(\cos t - \sin t)$

$x'(t) = 0$ when $\cos t = \sin t$. Therefore, $x'(t) = 0$ on $0 \le t \le 2\pi$ for $t = \frac{\pi}{4}$ and $t = \frac{5\pi}{4}$.

The candidates for the absolute minimum are at $t = 0, \frac{\pi}{4}, \frac{5\pi}{4}$, and 2π.

t	$x(t)$
0	$e^{0}\sin(0) = 0$
$\frac{\pi}{4}$	$e^{-\frac{\pi}{4}}\sin\left(\frac{\pi}{4}\right) > 0$
$\frac{5\pi}{4}$	$e^{-\frac{5\pi}{4}}\sin\left(\frac{5\pi}{4}\right) < 0$
2π	$e^{-2\pi}\sin(2\pi) = 0$

The particle is farthest to the left when $t = \frac{5\pi}{4}$.

$$5 : \begin{cases} 2 : x'(t) \\ 1 : \text{sets } x'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

(b) $x''(t) = -e^{-t}(\cos t - \sin t) + e^{-t}(-\sin t - \cos t)$

$= -2e^{-t}\cos t$

$Ax''(t) + x'(t) + x(t)$

$= A\left(-2e^{-t}\cos t\right) + e^{-t}(\cos t - \sin t) + e^{-t}\sin t$

$= (-2A + 1)e^{-t}\cos t$

$= 0$

Therefore, $A = \frac{1}{2}$.

$$4 : \begin{cases} 2 : x''(t) \\ 1 : \text{substitutes } x''(t),\ x'(t), \text{ and } x(t) \\ \quad \text{into } Ax''(t) + x'(t) + x(t) \\ 1 : \text{answer} \end{cases}$$

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$x(t) = e^{-t} \sin t$

$v(t) = e^{-t} \cos t + \sin t \, e^{-t} \cdot -1$

$v(t) = e^{-t}(\cos t - \sin t)$

$0 = e^{-t}(\cos t - \sin t)$

$\cos t = \sin t$

$t = \frac{\pi}{4}$ $t = \frac{5\pi}{4}$

t	$x(t)$
0	0
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2} e^{-\frac{\pi}{4}}$
$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2} e^{-\frac{5\pi}{4}}$
2π	0

$t = \frac{5\pi}{4}$. By closed interval test, $\frac{5\pi}{4}$ is the x coordinate of the absolute minimum. value of $x(t)$ on $[0, 2\pi]$

Continue problem 4 on page 11.

Work for problem 4(b)

$x(t) = e^{-t}\sin t \qquad x'(t) = e^{-t}\cos t - e^{-t}\sin t$

$x''(t) = -e^{-t}\sin t - \cos t\, e^{-t} - (e^{-t}\cos t - e^{-t}\sin t)$

$x''(t) = \cancel{-e^{-t}\sin t} - e^{-t}\cos t - e^{-t}\cos t \cancel{+ e^{-t}\sin t}$

$x''(t) = -2e^{-t}\cos t$

$A x''(t) + x'(t) + x(t) = 0$

$A(-2e^{-t}\cos t) + e^{-t}\cos t \cancel{- e^{-t}\sin t} \cancel{+ e^{-t}\sin t} = 0$

$-2Ae^{-t}\cos t + e^{-t}\cos t = 0$

$e^{-t}\cos t\,(-2A+1) = 0$

$A = \frac{-1}{-2}$

$\boxed{A = \frac{1}{2}}$

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GO ON TO THE NEXT PAGE.

4B₁

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$x'(t) = v(t) = e^{-t}\cos t + \sin t(e^{-t})(-1)$

$= e^{-t}\cos t - e^{-t}\sin t$

$= e^{-t}(\cos t - \sin t)$

$e^{-t}(\cos t - \sin t) = 0$

when $v(t)$ is negative, zero, then positive on the interval $[0, 2\pi]$

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Continue problem 4 on page 11.

$4B_2$

Work for problem 4(b)

$v(t) = e^{-t}(\cos t - \sin t)$

$x''(t) = e^{-t}(-\sin t - \cos t) + (\cos t - \sin t)(e^{-t})(-1)$

$e^{-t}(-\sin t - \cos t) - e^{-t}(\cos t - \sin t)$

$e^{-t}(-\cancel{\sin t} - \cos t - \cos t + \cancel{\sin t})$

$e^{-t}(-2\cos t)$

$A e^{-t}(-2\cos t) + e^{-t}(\cos t - \sin t) + e^{-t}\sin t = 0$

$Ae^{-t}(-2\cos t + \cos t - \cancel{\sin t} + \cancel{\sin t}) = 0$

$Ae^{-t}(\cos t) = 0 \quad [0, 2\pi]$

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$4C_1$

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$$x'(t) \text{ or } v(t) = e^{-t}(\cos t) + \sin t(e^{-t} \cdot -1)$$

$$= e^{-t}\cos t - e^{-t}\sin t$$

$$= e^{-t}(\cos t - \sin t)$$

Continue problem 4 on page 11.

Work for problem 4(b)

$x'(t) = e^{-t}(\cos t - \sin t)$

$x''(t) = e^{-t}(-\sin t - \cos t) + (\cos t - \sin t)(e^{-t})(-1)$

$= e^{-t}(-\sin t - \cos t) - e^{-t}(\cos t - \sin t)$

$= e^{-t}(-\cancel{\sin t} - \cos t - \cos t + \cancel{\sin t})$

$= e^{-t}(-2\cos t)$

$= -2e^{-t}\cos t$

$A(e^{-t}\sin t) + (e^{-t}(\cos t - \sin t)) - 2e^{-t}\cos t = 0$

$A(e^{-t}\cancel{\sin t} + \cancel{e^{-t}\cos t} - \cancel{e^{-t}\sin t} - \overset{1}{\cancel{2}}e^{-t}\cos t) = 0$

$A(-1e^{-t}\cos t) = 0$

AP® CALCULUS AB
2007 SCORING COMMENTARY

Question 4

Overview

This problem presented students with a function $x(t)$ describing the position of a particle at time t moving along the x-axis over a closed time interval. Part (a) asked for the time, with justification, when the particle was farthest to the left during this time interval. The first derivative of $x(t)$ was required to compute the time and complete the justification. Part (b) required students to substitute the function and the first and second derivatives of $x(t)$ into the equation $Ax''(t) + x'(t) + x(t) = 0$ to find the value of A. Students did not have to solve the differential equation to determine the value of A.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 3 points in part (a) and 3 points in part (b). In part (a) the student earned 2 points for $x'(t)$ and 1 point for setting $x'(t) = 0.$ In part (b) the student earned 2 points for $x''(t)$ and 1 point for the correct substitution. The student does not solve for A and thus did not earn the answer point.

Sample: 4C
Score: 4

The student earned 4 points: 2 points in part (a) and 2 points in part (b). In part (a) the student earned 2 points for $x'(t).$ In part (b) the student earned 2 points for $x''(t).$ The student incorrectly substitutes for $x''(t)$ and does not solve for A.

AP® CALCULUS AB
2007 SCORING GUIDELINES

Question 5

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.	$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{conclusion with reason} \end{cases}$
(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$ $\left.\frac{dV}{dt}\right\rvert_{t=5} = 4\pi(30)^2 2 = 7200\pi \text{ ft}^3/\text{min}$	$3 : \begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$
(c) $\int_0^{12} r'(t)\,dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$ $= 19.3$ ft $\int_0^{12} r'(t)\,dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.	$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$
(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t)\,dt$.	1 : conclusion with reason
Units of ft^3/min in part (b) and ft in part (c)	1 : units in (b) and (c)

5A

NO CALCULATOR ALLOWED

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

Work for problem 5(a)

$r(5)=30$

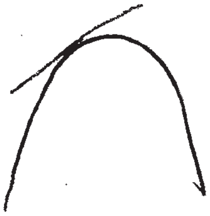
$y-30=2\cdot(x-5)$

$y-30=2\cdot(5.4-5)$

$y-30=.8$

$\boxed{y=30.8\text{ ft}}$

this estimate is greater than the actual Value because the graph of r(t) is concave down



Work for problem 5(b)

$V=\frac{4}{3}\pi r^3$

$\frac{dV}{dt}=\frac{4}{\cancel{3}}\cdot\cancel{3}\pi r^2\frac{dr}{dt}$

$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$

$\frac{dV}{dt}=4\pi\cdot 30^2\cdot 2$

$\frac{dV}{dt}=4\pi\cdot 900\cdot 2$

$\frac{dV}{dt}=$ 7200 π ft³/minute

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Continue problem 5 on page 13.

5A 2

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\int_0^{12} r'(t)\,dt = 2\cdot 4 + 3\cdot 2 + 2\cdot 1.2 + 4\cdot .6 + 1\cdot .5$$

$$= 8+6+2.4+2.4+.5$$

$$= 8+6+4.8+.5$$

$$= 14+5.3$$

$$= \boxed{19.3 \text{ feet}}$$

this is the change in the radius of the baloon from $t=0$ min to $t=12$ min

Work for problem 5(d)

the estimation is less than the actual Value because $r'(t)$ is decreasing on the interval $0<t<12$

GO ON TO THE NEXT PAGE.

5B

NO CALCULATOR ALLOWED

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

Work for problem 5(a)

$$L_n = f(a) + f'(a)(x-a)$$

$$= 30 + 2(x-5) = 30+2x-10 = 20+2x$$

Work for problem 5(b)

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = \cancel{\frac{3}{3}}\ 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dv}{dt} = 4\pi(30)^2(2) = 4\pi(900)(2) = \boxed{7200\pi\ \frac{feet^3}{minute}}$$

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Continue problem 5 on page 13.

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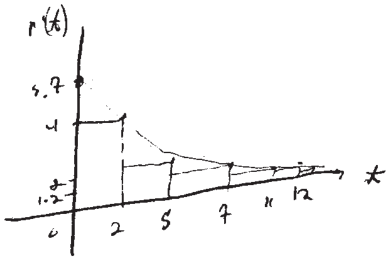
Work for problem 5(c)

$$RRAM = 1(.5) + \overset{4}{\cancel{3}}(.6) + 2(1.2) + 3(2) + 4(2) =$$

$$= .5 + 2.4 + 2.4 + 6 + 8 = \boxed{19.3 \text{ feet}}$$

$\int_0^{12} r'(t)dt$ is the sum of the area under the curve of $r'(t)$. It shows the radius of the balloon in feet ~~at~~ 12 minutes after it ~~goe~~ begins to expand.

Work for problem 5(d)



The approximation in part c is less than $\int_0^{12} r'(t)dt$ because the ~~areas of a~~ approximated areas within each subinterval fall below the graph of $r'(t)$ because $r'(t)$ is a decreasing function.

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NO CALCULATOR ALLOWED

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

Work for problem 5(a)

$R = 30 + 2 \times (t-5)$

$R = 30 + 2 \cdot 5 (t-5)$

$R = 30 + 10(t-5)$

$R = 10t - 20$

$R = 10(5.4) - 20$

$R = 34$ feet

This is less than the true value because $r'(t)$ is (+), so the radius is increasing, If r is measured at $t=5$, the measurement will be less than r when $r = 5.4$

Work for problem 5(b)

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 dr$

$\frac{dV}{dt} = 4\pi(30)^2(2)$

$\frac{dV}{dt} = 7200\pi$ ft³/min

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Continue problem 5 on page 13.

5C₂

NO CALCULATOR ALLOWED

Work for problem 5(c)

$S = 2(4+2+1.2+0.6+0.5)$

$S = 16.6$ feet

$\int_0^{12} r'(t)dt = r(t) =$ the radius of the balloon at time t.

Work for problem 5(d)

part (c) approximation is greater than $\int_0^{12} r'(t)dt$. Since the radius is increasing, the right value of each approximation is the highest value for each interval, so the approximation is greater.

GO ON TO THE NEXT PAGE.

Question 5

Overview

The problem presented students with a table of values for the rate of change of the radius of an expanding spherical balloon over a time interval of 12 minutes. Students were told that the radius was modeled by a twice-differentiable function whose graph was concave down. Part (a) asked students to use a tangent line approximation to estimate the radius of the balloon at a specific time and to determine if the estimate was greater than or less than the true value. This tested their ability to use the information about the concavity of the graph of the radius to make the appropriate conclusion about the behavior of the tangent line. In part (b) students had to handle the related rate of change of the volume, given information about the rate of change of the radius. In part (c) students had to recognize the definite integral as the total change, in feet, of the radius of the balloon from time $t = 0$ minutes to time $t = 12$ minutes and approximate the value of this integral using a right Riemann sum and the data in the table. Part (d) asked students to decide if this approximation was greater than or less than the true value of the definite integral. Again, they were required to use the information about the concavity of the graph of the radius to make the appropriate conclusion about the behavior of the graph of the derivative. Units of measure were important in parts (b) and (c).

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 3 points in part (b), 1 point in part (c), 1 point in part (d), and the units point. In part (a) the student finds the tangent line approximation correctly but does not compute the estimate nor state a conclusion or reason. Thus no points were earned. In part (b) the student correctly finds $\frac{dV}{dt}$ using the chain rule and is therefore eligible for the answer point. This answer is also correct. In part (c) the student finds the correct approximation using a right Riemann sum but fails to provide a correct explanation—this integral represents the *change* in radius, not the radius, after 12 minutes. In part (d) the student correctly identifies the reason that the approximation is less than the actual value: $r'(t)$ is decreasing. The student earned the units point.

Sample: 5C
Score: 4

The student earned 4 points: 3 points in part (b) and the units point. In part (a) the student calculates the estimate for $r(5.4)$ incorrectly and states that $r'(t)$ is positive rather than decreasing. In part (b) the student earned all 3 points for correct use of the chain rule and the correct calculation of the result. In part (c) the student makes an error in calculating the right Riemann sum and does not refer to the *change* in radius. In part (d) the student states that the approximation is greater than the definite integral rather than less. The student earned the units point.

Question 6

Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

(a) Find $f'(x)$ and $f''(x)$.

(b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k, determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.

(c) For a certain value of the constant k, the graph of f has a point of inflection on the x-axis. Find this value of k.

(a) $f'(x) = \dfrac{k}{2\sqrt{x}} - \dfrac{1}{x}$

$f''(x) = -\dfrac{1}{4}kx^{-3/2} + x^{-2}$

$$2 : \begin{cases} 1 : f'(x) \\ 1 : f''(x) \end{cases}$$

(b) $f'(1) = \dfrac{1}{2}k - 1 = 0 \Rightarrow k = 2$

When $k = 2$, $f'(1) = 0$ and $f''(1) = -\dfrac{1}{2} + 1 > 0$.

f has a relative minimum value at $x = 1$ by the Second Derivative Test.

$$4 : \begin{cases} 1 : \text{sets } f'(1) = 0 \text{ or } f'(x) = 0 \\ 1 : \text{solves for } k \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

(c) At this inflection point, $f''(x) = 0$ and $f(x) = 0$.

$$f''(x) = 0 \Rightarrow \frac{-k}{4x^{3/2}} + \frac{1}{x^2} = 0 \Rightarrow k = \frac{4}{\sqrt{x}}$$

$$f(x) = 0 \Rightarrow k\sqrt{x} - \ln x = 0 \Rightarrow k = \frac{\ln x}{\sqrt{x}}$$

Therefore, $\dfrac{4}{\sqrt{x}} = \dfrac{\ln x}{\sqrt{x}}$

$\Rightarrow 4 = \ln x$

$\Rightarrow x = e^4$

$\Rightarrow k = \dfrac{4}{e^2}$

$$3 : \begin{cases} 1 : f''(x) = 0 \text{ or } f(x) = 0 \\ 1 : \text{ equation in one variable} \\ 1 : \text{ answer} \end{cases}$$

6A,

Work for problem 6(a)

$f(x) = k\sqrt{x} - \ln x$

$f'(x) = \frac{k}{2}x^{-1/2} - \frac{1}{x}$

$f''(x) = \frac{-k}{4}x^{-3/2} + x^{-2}$

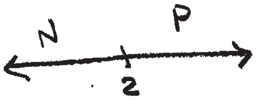
Work for problem 6(b)

$f'(x) = \frac{k}{2}x^{-1/2} - \frac{1}{x}$

$0 = \frac{k}{2}(1)^{-1/2} - \frac{1}{1}$

$1 = \frac{k}{2}$; $k = 2$

for $k = 2$, f has a critical point at $x = 1$.



FOR $k = 2$, f has a relative minimum at $x = 1$, since the derivative changes from negative to positive at that location, which means the function changes from decreasing to increasing, creating a local minimum

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

6A₂

Work for problem 6(c)

$f''(x) = \frac{-k}{4} x^{-3/2} + x^{-2}$

$0 = \frac{-k}{4} x^{-3/2} + x^{-2}$

$x^{-2} = \frac{k}{4} x^{-3/2}$

$4x^{-2} = k x^{-3/2}$

$4x^{-1/2} = k$

$0 = 4x^{-1/2} (x^{1/2}) - \ln x$

$0 = 4 - \ln x$

$4 = \ln x$

$x = e^4$

$(e^4)^{-2} = \frac{k}{4} (e^4)^{-3/2}$

$e^{-8} = \frac{k}{4} e^{-6}$

$e^{-2} = \frac{k}{4}$

$k = \frac{4}{e^2}$

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6B1

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Work for problem 6(a)

$f(x) = k(x)^{1/2} - \ln x$

$f'(x) = \frac{kx^{-1/2}}{2} - \frac{1}{x}$

$f''(x) = \frac{-kx^{-3/2}}{4} - \frac{-1}{x^2}$

$= \frac{-Kx^{-3/2}}{4} + \frac{1}{x^2}$

Work for problem 6(b)

x	.5	1	4
$\frac{1}{\sqrt{.5}} - 2$	1	0	$\frac{1}{4}$

$\frac{1}{2} - \frac{1}{4}$

− 0 +

$f(x) = 0 = \frac{Kx^{-1/2}}{2} - \frac{1}{x}$

~~f(1)~~ $= 0 = \frac{K \cdot 1}{2} - 1$

$1 = \frac{K}{2}$

$\boxed{2 = K}$

relitive minimum at x = 1 because the function is decreasing to the left of the critical point and increasing on the right.

$\frac{16}{.1}$

.21

.05

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

6B$_2$

Work for problem 6(c)

$$0 = \frac{-k x^{-3/2}}{4} + \frac{1}{x^2}$$

$$-\frac{1}{x^2} = \frac{-k x^{-3/2}}{4}$$

$$\frac{4}{-x^2} = -k x^{-3/2}$$

$$\frac{4}{x^2} = k x^{-3/2}$$

$$\frac{4 x^{3/2}}{x^2} = k$$

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$6C_1$

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Work for problem 6(a)

$kx^{\frac{1}{2}} - \ln x$

$f'(x) = \frac{1}{2}kx^{-1/2} - \frac{1}{x}$

$f''(x) = -\frac{1}{4}kx^{-3/2} + x^{-2}$

Work for problem 6(b)

critical point at x=1 what value of constant

$k(1)^{1/2} - \ln(1)$

$k\,1/2 =$

$\boxed{\text{relative minimum}}$

$k = \frac{f(x)}{1/2}$

Continue problem 6 on page 1

6C2

Work for problem 6(c)

$$0 = -\frac{1}{4}KX^{-3/2} + X^{-2}$$

$$0 = -\frac{1}{4}K\cancel{X}^{-3/2} + \cancel{X}^{-2}$$

$$K = -\frac{1}{4}X^{-3/2} + X^{-2}$$

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AP® CALCULUS AB
2007 SCORING COMMENTARY

Question 6

Overview

This problem presented students with a function that contained a parameter k. In part (a) students had to find the first and second derivatives of the function, making the distinction between the parameter and the variable. Parts (b) and (c) involved finding values of k so that the function or its graph would satisfy certain properties. In part (b) students had to find the value of k for which the function had a critical point at $x = 1$, and then determine whether the function had a relative minimum, relative maximum, or neither at this critical point. In part (c) students were told that the graph of the function had a point of inflection on the x-axis for a certain value of k and were asked to find that value. The x-coordinate of the point of inflection was not given so students had to write and then solve two nonlinear equations to determine the value of k (and possibly the value of x). Because the problem stated that a point of inflection existed, students were not required to justify that the k value they found actually produced a point of inflection of the graph of the function.

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 6 points: 2 points in part (a), 3 points in part (b), and 1 point in part (c). In part (a) the student has the correct first and second derivatives, which earned the points. In part (b) the student earned the points for setting the first derivative equal to 0, solving for k, and declaring the critical point a minimum. The justification point was not earned because the student states that the function decreases and then increases after 1. In part (c) the student earned the point for setting the second derivative equal to 0.

Sample: 6C
Score: 3

The student earned 3 points: 2 points in part (a) and 1 point in part (c). In part (a) the student has the correct first and second derivatives, which earned the points. In part (c) the student earned the point for setting the second derivative equal to 0.